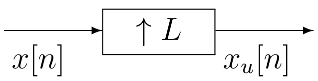
PhD Written Preliminary Exam: Problem 3 (Signal Processing)

November 7, 2015

Instructions:

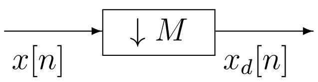
1. Answer all problems.
2. There are 2 problems with a total of 40 points.
3. Upsampling and downsampling operations are defined mathematically in the sequence domain (see below). You do not have to derive the relationships in the frequency domain. For example, if you know the relationship between $X_u\left(e^{j\omega}\right)$ and $X\left(e^{j\omega}\right)$, then simply use it if it is pertinent to your answer.
4. Show work for partial credit.

Upsampling/Downsampling:



(Upsampling)

$$x_u[n] = \begin{cases} x[n/L] & \text{for } n = 0, \pm L, \pm 2L, \ldots, \\ 0 & \text{otherwise.} \end{cases}$$

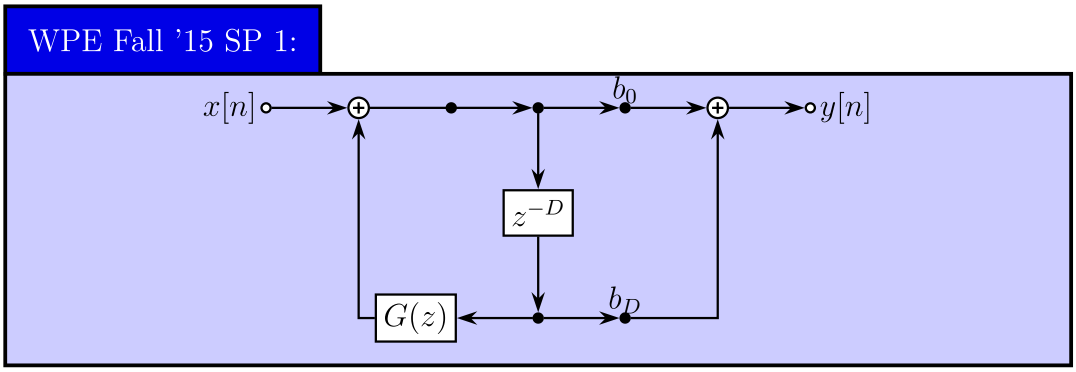


(Downsampling)

$$\begin{aligned} x_d[n] &= x[Mn] \\ &= p[Mn]x[Mn]. \end{aligned}$$

Where $p[n]$ is a periodic unit pulse with period M.

1. Consider the following system:



For parts (a) – (c), assume $b_0 = -a, b_D = 1$ and $G(z) = a$ with $0 < a < 1$

(a) Derive the difference equation for the output, $y[n]$. [5 points]

(b) Derive the relationship between $Y(e^{j\omega})$ and $X(e^{j\omega})$ and sketch the magnitude response of the filter. You will not need specific values for D and a, but you may assume $D = 8$ and $a = 0.75$. [5 points]

(c) Show that the unit pulse response[1] can be written in the form:

$$h[n] = c_1\delta[n] + c_2\left(a\delta[n-D] + a^2\delta[n-2D] + a^3\delta[n-3D] + \cdots\right)$$

Explain why this filter is used to model reverberations in audio applications. [5 points]

(d) Now assume $b_0 = 1, b_D = 0$ and $G(z) = a(1+z^{-1}+z^{-2})$. Derive an expression for $h[n]$ and explain why the unit pulse response becomes denser for large values of n. [5 points]

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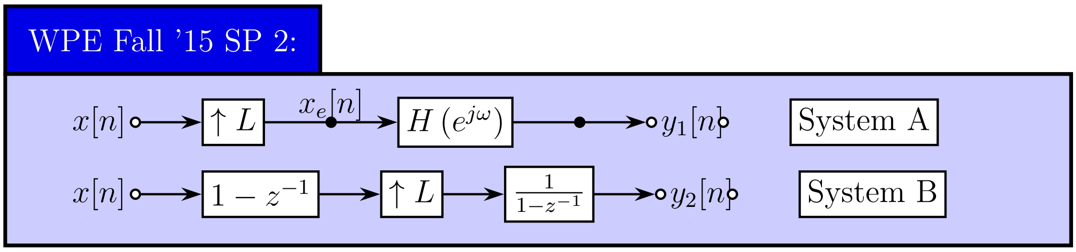
[1]Some authors refer to this as the impulse response.

2. Consider the two upsampling and interpolation systems shown in the figure:



In System A, the frequency response of the filter is given by

$$H\left(e^{j\omega}\right) = \begin{cases} L, & |\omega| < \frac{\pi}{L} \\ 0, & \frac{\pi}{L} < |\omega| \leq \pi. \end{cases}$$

Assume

$$x[n] = \frac{\sin(\omega_N n)}{\pi n}, \qquad \omega_N < \pi$$

as the input for both systems and answer the following questions.

(a) Sketch $|X_e(e^{j\omega})|$ and $|Y_1(e^{j\omega})|$. You may assume $L = 3$. [5 points]

(b) Sketch $|Y_2(e^{j\omega})|$. You may assume $L = 3$. Comment on the nature of imperfection in signal interpolation. [10 points]

(c) Comment on the merits and shortcomings of the two systems in terms of the feasibility and cost of implementation. [5 points]